

1. Výraz $ab + c$ vyjádři jako mnohočlen s proměnou x , který je uspořádaný sestupně, je-li:

a) $a = x + 1 \quad b = x^2 - 1 \quad c = x^3 + 1$

$$ab + c = (x + 1)(x^2 - 1) + (x^3 + 1) = x^3 - x + x^2 - 1 + x^3 + 1 = 2x^3 + x^2 - x$$

b) $a = 2x - 1 \quad b = 2x - 1 \quad c = x$

$$ab + c = (2x - 1)(2x - 1) + (x) = 4x^2 - 4x + 1 + x = 4x^2 - 3x + 1$$

c) $a = b = c = 3x - 2$

$$ab + c = (3x - 2)(3x - 2) + (3x - 2) = 9x^2 - 12x + 4 + 3x - 2 = 9x^2 - 9x + 2$$

d) $a = x - 2 \quad b = 3 - x \quad c = x^2 + 1$

$$ab + c = (x - 2)(3 - x) + (x^2 + 1) = 3x - x^2 - 6 + 2x + x^2 + 1 = 5x - 5 = 5(x - 1)$$

2. Urči mnohočlen, který je nutno přičíst k mnohočlenu $(x + y)^2 + r^2$, abychom dostali mnohočlen $(x + y + r)^2$.

$$(x + y)^2 + r^2 = x^2 + 2xy + y^2 + r^2$$

$$(x + y + r)^2 = (x + y + r)(x + y + r) = x^2 + xy + xr + yx + y^2 + yr + rx + ry + r^2 =$$

$$= x^2 + y^2 + r^2 + 2xy + 2xr + 2yr$$

$$(x + y + r)^2 - [(x + y)^2 + r^2] = (x^2 + y^2 + r^2 + 2xy + 2xr + 2yr) - (x^2 + 2xy + y^2 + r^2) = 2xr + 2yr$$

Musíme připočíst výraz $2xr + 2yr$.

3. Zjednoduš výraz:

a) $x(x^2 + xy + y^2) - y(x^2 - xy - y^2) - x(x^2 + 2y^2) = x^3 + x^2y + xy^2 - (x^2y - xy^2 - y^3) - (x^3 + 2xy^2) =$
 $= x^3 + x^2y + xy^2 - x^2y + xy^2 + y^3 - x^3 - 2xy^2 = y^3$

$$ab(c + d) - ac(b + d) + ad(b - c) + bc(a + d) - bd(a - c) + cd(a - b) =$$

b) $= abc + abd - abc - acd + abd - acd + abc + bcd - abd + bcd + acd - bcd =$
 $= abc + abd - acd + bcd$

c) $4a(5b - 2a) - 4(7a^2 - 3ab) - 2a(3a - 3b) = 20ab - 8a^2 - 28a^2 + 12ab - 6a^2 + 6ab =$
 $= 38ab - 42a^2 = 2a(19b - 21a)$

d) $1,4x(0,5x - 0,3y) - 5(0,4y^2 - 4xy) + 0,2y(8y - 5x) = 0,7x^2 - 0,42xy - 2y^2 + 20xy + 1,6y^2 - xy =$
 $= 0,7x^2 + 18,58xy - 0,4y^2$

e) $r^3(r^2 + 3) + r^2(r^3 + r^2) - r^3(r + 1) = r^5 + 3r^3 + r^5 + r^4 - r^4 - r^3 = 2r^5 + 2r^3 = 2r^3(r^2 + 1)$

f) $x^3(x + y^3) - (xy)^3 + (2x^2)^3 = x^4 + x^3y^3 - x^3y^3 + 8x^6 = x^4 + 8x^6 = x^4(1 + 8x^2)$

g) $(a^2b^3)^2 + (2a^2)^2y^2 - (a^2y)^2 - a^4(b^6 + 1) = a^4b^6 + 4a^4y^2 - a^4y^2 - a^4b^6 - a^4 = 3a^4y^2 - a^4 =$
 $= a^4(3y^2 - 1)$

4. Zjednoduš výraz:

a) $(2x - 1)^3 - (x - 2)^3 = (8x^3 - 12x^2 + 6x - 1) - (x^3 - 6x^2 + 12x - 8) = 7x^3 - 6x^2 - 6x + 7$

$$(3x+y)^3 - (9x^2 + 6xy + y^2)(3x-y) =$$

$$\begin{aligned} \text{b) } &= 27x^3 + 27x^2y + 9xy^2 + y^3 - (27x^3 + 18x^2y + 3xy^2 - 9x^2y - 6xy^2 - y^3) = \\ &= 18x^2y + 12xy^2 + 2y^3 \end{aligned}$$

$$\text{c) } (a+2)^3 - 3(a+2)^2(a+1) + 3(a+2)(a+1)^2 - (a+1)^3 = [(a+2) - (a+1)]^3 = (1)^3 = 1$$

$$(a^2-1)^3 - (a^2-1)(a^2+1)^2 + 2a^2(a^2-2) + a^4(a^4+2) =$$

$$\begin{aligned} \text{d) } &= (a^6 - 3a^4 + 3a^2 - 1) - (a^2 - 1)(a^4 + 2a^2 + 1) + 2a^4 - 2a^2 + a^8 + 2a^4 = \\ &= a^8 + a^6 + a^4 + a^2 - 1 - (a^6 + 2a^4 + a^2 - a^4 - 2a^2 - 1) = a^8 \end{aligned}$$

$$\text{e) } (2x-1)^3(2x+1)^3 = [(2x-1)(2x+1)]^3 = [4x^2-1]^3 = 64x^6 - 48x^4 + 12x^2 - 1$$

$$\begin{aligned} \text{f) } &(a^2 - ab + b^2)^3(a+b)^3 = [(a^2 - ab + b^2)(a+b)]^3 = [a^3 + a^2b - a^2b - ab^2 + b^2a + b^3]^3 = \\ &= (a^3 + b^3)^3 \end{aligned}$$

$$(a+b)^2 - (a-b)^2 + (ab+1)^2 - (ab-1)^2 =$$

$$\begin{aligned} \text{g) } &= a^2 + 2ab + b^2 - (a^2 - 2ab + b^2) + a^2b^2 + 2ab + 1 - (a^2b^2 - 2ab + 1) = \\ &= 4ab + 4ab = 8ab \end{aligned}$$

$$\text{h) } [(p+1)^2 - (p-1)^2]^2 = [p^2 + 2p + 1 - (p^2 - 2p + 1)]^2 = (4p)^2 = 16p^2$$

$$\begin{aligned} \text{i) } &[(2x^2 - 3y^3)^2 + (3x^2 + 2y^3)^2]^2 = [4x^4 - 12x^2y^3 + 9y^6 + (9x^4 + 12x^2y^3 + 4y^6)]^2 = \\ &= (13x^4 + 13y^6)^2 = [13(x^4 + y^6)]^2 = 13^2(x^8 + 2x^4y^6 + y^{12}) = 169x^8 + 338x^4y^6 + 169y^{12} \end{aligned}$$

5. O kolik se zvětší hodnota výrazu $(a+b+1)^2$, zvětší-li se číslo a o 1?

Spočtu výraz

$$\begin{aligned} &([a+1] + b + 1)^2 - (a + b + 1)^2 = (a + b + 2)^2 - (a + b + 1)^2 = (a + b + 2)(a + b + 2) - (a + b + 1)(a + b + 1) = \\ &= a^2 + ab + 2a + ba + b^2 + 2b + 2a + 2b + 4 - (a^2 + ab + a + ba + b^2 + b + a + b + 1) = \\ &= a^2 + 2ab + b^2 + 4a + 4b + 4 - (a^2 + 2ab + b^2 + 2a + 2b + 1) = 2a + 2b + 3 \end{aligned}$$

6. Stanov podmínky a děl:

$$(6x^2 - 11x - 10) : (3x + 2) = 2x - 5$$

$$-(6x^2 + 4x)$$

$$\text{a) } -15x - 10$$

$$-(-15x - 10)$$

$$0$$

$$(a^3 - b^3) : (a - b) = a^2 + ab + b^2$$

$$-(a^3 - a^2b)$$

$$a^2b - b^3$$

$$\text{b) } -(a^2b - ab^2)$$

$$ab^2 - b^3$$

$$-(ab^2 - b^3)$$

$$0$$

$$(c^3 + c^2 - 11c - 15) : (c + 3) = c^2 - 2c - 5$$

$$-(c^3 + 3c^2)$$

$$\text{c) } -2c^2 - 11c - 15$$

$$-(-2c^2 - 6c)$$

$$-5c - 15$$

$$-(-5c - 15)$$

$$(9y^4 + 26y^2 + 25) : (3y^2 - 2y + 5) = 3y^2 + 2y + 5$$

$$-(9y^4 - 6y^3 + 15y^2)$$

$$\text{d) } 6y^3 + 11y^2 + 25$$

$$-(6y^3 - 4y^2 + 10y)$$

$$15y^2 - 10y + 25$$

$$-(15y^2 - 10y + 25)$$

$$(x^4 - 8x^3 + 16x^2 - 7x - 2) : (x^2 - 3x + 2) = x^2 - 5x - 1$$

$$-(x^4 - 3x^3 + 2x^2)$$

$$\text{e) } -5x^3 + 14x^2 - 7x - 2$$

$$-(-5x^3 + 15x^2 - 10x)$$

$$-x^2 + 3x - 2$$

$$-(-x^2 + 3x - 2)$$

$$\begin{aligned} & (11p^3 - 32 + 19p^2 + 3p^4 - 28p) : (4 - 3p) = \\ & (3p^4 + 11p^3 + 19p^2 - 28p - 32) : (-3p + 4) = -p^3 - 5p^2 - 13p - 8 \\ & -(3p^4 - 4p^3) \end{aligned}$$

$$\begin{aligned} & 15p^3 + 19p^2 - 28p - 32 \\ \text{f)} \quad & -(15p^3 - 20p^2) \\ & 39p^2 - 28p - 32 \\ & -(39p^2 - 52p) \\ & 24p - 32 \\ & -(24p - 32) \end{aligned}$$

$$\begin{aligned} & (x^5 + 4x^4 + 4x^3 - x - 2) : (x + 2) = x^4 + 2x^3 - 1 \\ & -(x^5 + 2x^4) \end{aligned}$$

$$\begin{aligned} & 2x^4 + 4x^3 - x - 2 \\ \text{g)} \quad & -(2x^4 + 4x^3) \\ & -x - 2 \\ & -(-x - 2) \end{aligned}$$

$$\begin{aligned} & (x^7 - x^5 - x^4 + 1) : (x^2 - 1) = x^5 - x^2 - 1 \\ & -(x^7 - x^5) \end{aligned}$$

$$\begin{aligned} & -x^4 + 1 \\ \text{h)} \quad & -(-x^4 + x^2) \\ & -x^2 + 1 \\ & -(-x^2 + 1) \end{aligned}$$

$$\begin{aligned} & (x^8 + x^7 - x^6 + x^5 + x^3 + x^2 - x + 1) : (x^5 + 1) = x^3 + x^2 - x + 1 \\ & -(x^8 + x^3) \end{aligned}$$

$$\begin{aligned} & x^7 - x^6 + x^5 + x^2 - x + 1 \\ \text{i)} \quad & -(x^7 + x^2) \\ & -x^6 + x^5 - x + 1 \\ & -(-x^6 - x) \\ & x^5 + 1 \\ & -(x^5 + 1) \end{aligned}$$

$$(3y^4 - 4y^3 - 7y^2 + 8y + 2) : (4y^2 - 8) = \frac{3}{4}y^2 - y - \frac{1}{4}$$

$$-(3y^4 - 6y^2) \\ -4y^3 - y^2 + 8y + 2$$

$$\text{j) } -(4y^3 + 8y) \\ -y^2 + 2 \\ -(-y^2 + 2) \\ 0$$

$$(2x^4 - 7x^3 - 2x^2 + 10x) : (2x^2 - 3x + 2) = x^2 - 2x - 5 + \frac{-x+10}{2x^2-3x+2}$$

$$-(2x^4 - 3x^3 + 2x^2) \\ -4x^3 - 4x^2 + 10x$$

$$\text{k) } -(-4x^3 + 6x^2 - 4x) \\ -10x^2 + 14x \\ -(-10x^2 + 15x - 10) \\ -x + 10$$

7. Rozlož mnohočleny na součin:

$$\text{a) } x(a+b)^2 + x^2(a+b) = x(a+b)[(a+b)+x] = x(a+b)(a+b+x)$$

$$\text{b) } ax^5 - 2a^2x^4 + a^3x^3 = ax^3(x^2 - 2ax + a^2) = ax^3(x-a)^2$$

$$\text{c) } 8b^2 - 18c^2 = 2(4b^2 - 9c^2) = 2[(2b)^2 - (3c)^2] = 2(2b-3c)(2b+3c)$$

$$\text{d) } 9p^4(a-b) - 25q^2(a-b) = (a-b)[(3p^2)^2 - (5q)^2] = (a-b)(3p^2+5q)(3p^2-5q)$$

$$\text{e) } 9x^2 - 6xy + y^2 - z^2 = [(3x)^2 - 6xy + y^2] - z^2 = (3x-y)^2 - z^2 = [(3x-y)-z][(3x-y)+z] = \\ = (3x-y-z)(3x-y+z)$$

$$\text{f) } (a-b)x^4 + (b-a)x^2 = x^2[(a-b)x^2 - (a-b)] = (a-b)x^2[x^2-1] = (a-b)x^2(x+1)(x-1)$$

$$\begin{aligned} & (a^2 + b^2 - c^2)^2 - 4a^2b^2 = (a^2 + b^2 - c^2)^2 - (2ab)^2 = (a^2 + b^2 - c^2 - 2ab)(a^2 + b^2 - c^2 + 2ab) = \\ \text{g) } & = (a^2 - 2ab + b^2 - c^2)(a^2 + 2ab + b^2 - c^2) = [(a-b)^2 - c^2][(a+b)^2 - c^2] = \\ & = [(a-b)-c][(a-b)+c][(a+b)-c][(a+b)+c] = \\ & = (a-b-c)(a-b+c)(a+b-c)(a+b+c) \end{aligned}$$

$$\text{h) } 2a^5 - 2a = 2a(a^4 - 1) = 2a[(a^2)^2 - 1] = 2a(a^2 - 1)(a^2 + 1) = 2a(a-1)(a+1)(a^2 + 1)$$

$$\begin{aligned}
 & a(p-q+1)(ax^2+b) + b(p-q+1)(bx^2-a) + 2abx^2(p-q+1) = \\
 \text{i)} \quad & = (p-q+1)[a(ax^2+b) + b(bx^2-a) + 2abx^2] = (p-q+1)[a^2x^2 + ab + b^2x^2 - ba + 2abx^2] = \\
 & = (p-q+1)[a^2x^2 + b^2x^2 + 2abx^2] = (p-q+1)x^2(a^2 + 2ab + b^2) = \\
 & = (p-q+1)x^2(a+b)^2
 \end{aligned}$$

$$\begin{aligned}
 \text{j)} \quad & (r+s)^4 - r^4 = [(r+s)^2]^2 - [r^2]^2 = [(r+s)^2 + r^2][(r+s)^2 - r^2] = \\
 & = [r^2 + 2rs + s^2 + r^2][(r+s) - r][(r+s) + r] = (2r^2 + 2rs + s^2)s(2r+s)
 \end{aligned}$$

$$\begin{aligned}
 \text{k)} \quad & xz - yz - x^2 + 2xy - y^2 = z(x-y) - (x^2 - 2xy + y^2) = z(x-y) - (x-y)^2 = \\
 & = (x-y)[z - (x-y)] = (x-y)(z - x + y)
 \end{aligned}$$

$$\text{l)} \quad x^3 - 3x^2 - 4x + 12 = x^2(x-3) - 4(x-3) = (x-3)(x^2 - 4) = (x-3)(x-2)(x+2)$$

$$\begin{aligned}
 \text{m)} \quad & 2k^4 - k^3 + k - 2 = 2k^4 - 2 - k^3 + k = 2(k^4 - 1) - k(k^2 - 1) = 2([k^2]^2 - 1) - k(k^2 - 1) = \\
 & = 2(k^2 - 1)(k^2 + 1) - k(k^2 - 1) = (k^2 - 1)[2(k^2 + 1) - k] = (k-1)(k+1)(2k^2 - k + 2)
 \end{aligned}$$

$$\begin{aligned}
 \text{n)} \quad & y^4 - 2y^3 + 2y^2 - 2y + 1 = y^4 - 2y^3 + y^2 + y^2 - 2y + 1 = y^2(y^2 - 2y + 1) + (y^2 - 2y + 1) = \\
 & = y^2(y-1)^2 + (y-1)^2 = (y-1)^2(y^2 + 1) = (y-1)(y-1)(y^2 + 1)
 \end{aligned}$$

$$\begin{aligned}
 \text{o)} \quad & 2h^2 + h - 1 = h^2 + h + h^2 - 1 = h(h+1) + (h^2 - 1) = h(h+1) + (h+1)(h-1) = (h+1)[h + (h-1)] = \\
 & = (h+1)(2h-1)
 \end{aligned}$$

$$\text{p)} \quad 27r^4 - r = r(27r^3 - 1) = r[(3r)^3 - 1] = r(3r-1)(9r^2 + 3r + 1)$$

$$\begin{aligned}
 \text{q)} \quad & a^3 + 3a^2 + 4a + 2 = a^3 + 3a^2 + 2a + 2a + 2 = a(a^2 + 3a + 2) + 2(a+1) = a(a+1)(a+2) + 2(a+1) = \\
 & = (a+1)[a(a+2) + 2] = (a+1)(a^2 + 2a + 2)
 \end{aligned}$$

$$\text{r)} \quad x^2 - x - 72 = x^2 - 9x + 8x - 72 = x(x-9) + 8(x-9) = (x-9)(x+8)$$

$$\begin{aligned}
 \text{s)} \quad & x^3 + x^2 - 42x = x(x^2 + x - 42) = x[x^2 + 7x - 6x - 42] = x[x(x+7) - 6(x+7)] = \\
 & = x(x+7)(x-6)
 \end{aligned}$$

$$\text{t)} \quad 4x^2 - 8x + 3 = 4x^2 - 6x - 2x + 3 = 2x(2x-3) - (2x-3) = (2x-3)(2x-1)$$

$$\text{u)} \quad 3a^2 + 5a - 2 = 3a^2 + 6a - a - 2 = 3a(a+2) - (a+2) = (a+2)(3a-1)$$

$$\text{v)} \quad 2y^2 + 3y + 1 = 2y^2 + 2y + y + 1 = 2y(y+1) + (y+1) = (y+1)(2y+1)$$

$$\begin{aligned} \text{w)} \quad & x^2 + (a-3)x + 2(1-a) = x^2 + ax - 3x - 2a + 2 = x^2 + ax - x - 2x - 2a + 2 = \\ & = x(x+a-1) - 2(x+a-1) = (x+a-1)(x-2) \end{aligned}$$

$$\text{x)} \quad x^6 - y^6 = (x^3)^2 - (y^3)^2 = (x^3 - y^3)(x^3 + y^3) = (x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)$$

$$\begin{aligned} \text{y)} \quad & (x^2 - 2x + 3)^2 - (x^2 - 2x - 3)^2 = \left[(x^2 - 2x + 3) - (x^2 - 2x - 3) \right] \left[(x^2 - 2x + 3) + (x^2 - 2x - 3) \right] = \\ & 6(2x^2 - 4x) = 6 \cdot 2x(x-2) = 12x(x-2) \end{aligned}$$

$$\begin{aligned} \text{z)} \quad & 2x^4 + x^3 + 4x^2 + x + 2 = 2x^4 + x^3 + 2x^2 + 2x^2 + x + 2 = x^2(2x^2 + x + 2) + (2x^2 + x + 2) = \\ & = (2x^2 + x + 2)(x^2 + 1) \end{aligned}$$

$$\begin{aligned} \text{ž)} \quad & t^3 + 3t^2 + 4t + 2 = t^3 + 3t^2 + 2t + 2t + 2 = t(t^2 + 3t + 2) + 2(t+1) = t(t+1)(t+2) + 2(t+1) = \\ & = (t+1)[t(t+2) + 2] = (t+1)(t^2 + 2t + 2) \end{aligned}$$